

Level 2 Further Maths Scheme of Work:

Half Term 1: Number and Algebra I, Algebra II, Algebra III

Number and Algebra I – Prior Knowledge Review (7 hours)					
Topic (priority over L.O)	Specification link	Prior knowledge	Teaching points		Free resources
Numbers and the number system	1.1: Number Understand and use the correct hierarchy of operations. Understand and use decimals, fractions and percentages. Understand rounding and give answers to an appropriate degree of accuracy.	GCSE	Encourage students to share different methods; non-calculator and calculator.	Knowledge and use of numbers and the number system including fractions, decimals, percentages, ratio, proportion and order of operations are expected	Digits
Simplifying expressions	2.1: The basic processes of algebra	GCSE	Students should understand when and why algebraic expressions can be combined.	Knowledge and use of basic skills in manipulative algebra including use of the associative, commutative and distributive laws, are expected	simplify
Solving linear equations	2.14: Solution of linear equations	2.1	Encourage students to share different methods. Finding a deliberate error in a step by step solution is a useful activity.	Solutions of quadratics to include solution by factorisation, by graph, by completing the square or by formula. Problems will be set in a variety of contexts, which result in the solution of linear or quadratic equations.	Solve linear equations
Algebra and number	1.1: Number Understand and use ratio and proportion. Understand and use decimals, fractions and percentages. 1.4 – Textbook – Ratio and percentages with unknowns.	GCSE	An appreciation that $a\%$ of b is the same as $b\%$ of a can be useful. Students may find it useful to change ratio problems into fractional problems using $x: y = a: b \Leftrightarrow \frac{x}{y} = \frac{a}{b}$	Knowledge and use of numbers and the number system including fractions, decimals, percentages, ratio, proportion and order of operations are expected	Counting fractions
Expanding brackets	2.6: Expanding brackets and collecting like terms	2.1	When expanding two brackets with 3+ terms, students should be encouraged to adopt a systematic approach to avoid omissions or duplications.	Expand and simplify $(y^2 - 2y + 3)(2y - 1) - 2(y^3 - 3y^2 + 4y - 2)$	Expanding brackets
Manipulating surds	1.3: Manipulation of surds, including rationalising the denominator	1.1	Use $\sqrt{3^2 + 4^2} \neq 3 + 4$ to demonstrate that $\sqrt{a^2 + b^2}$ does not simplify to $a + b$. Use $\sqrt{3} \times \sqrt{3} = 3$ to explain that $\sqrt{x} \times \sqrt{x} = x$ Explain that the square root sign gives the positive root only, e.g. $\sqrt{4} = 2$, but not -2 . However, the solution to $x^2 = 4$ is $x = \pm 2$	Manipulation of surds, including rationalising the denominator The use of surds in exact calculations Write $\sqrt{200} - \sqrt{72} + 3\sqrt{162}$ in the form of $a\sqrt{2}$ Rationalise and simplify $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$ Write your answer in the form $a + b\sqrt{3}$ where a and b are integers	Surds
The product rule for counting	1.2: The product rule for counting	GCSE	When solving such problems, it is sometimes easier to first solve similar a problem with smaller numbers. Then, if necessary, all outcomes can be listed.		Product rule
The binomial expansion	2.7: Expand $(a + b)^n$ for positive integer n	2.6	Mistakes often occur when students simplify their expansion prematurely.	Expand and simplify $(5x + 2)^3$ Use Pascal's triangle to work out the coefficient of x^3 in the expansion of $(3 + 2x)^5$	Binomial expansion

2. Algebra II – Further Algebraic Methods (5 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
Factorising	2.8: Factorising	2.6	A quick recap of expanding brackets may be useful first.	Factorise fully $(2x + 3)^2 - (2x - 5)^2$ Factorise $15x^2 \pm 34xy - 16y^2$ Factorise fully $x^4 - 25x^2$	Difference of two squares
Rearranging formulae	2.10: Use and manipulation of formulae	Solution of linear equations	Sometimes useful to replace the letters with numbers to illustrate the processes involved.	Rearrange $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to make v the subject	Rearranging formulae
Simplifying algebraic fractions	2.9: Manipulation of rational expressions: Use of $+$ $-$ \times \div for algebraic fractions with denominators being numeric, linear or quadratic	2.8	First remind students of the principles involved when adding, subtracting, multiplying, dividing and simplifying fractions. Discuss why $\frac{6+8}{10}$ is the same as $\frac{3+4}{5}$ but $\frac{6 \times 8}{10}$ is not the same as $\frac{3 \times 4}{5}$	Simplify $\frac{5}{x+2} - \frac{3}{2x-1}$	Algebraic fractions Difference
Solving linear equations involving fractions				Simplify $\frac{x^3+2x^2+x}{x^2+x}$ Simplify $\frac{5x^2-14x-3}{4x^2-25} \div \frac{x-3}{4x^2+10x}$	
Completing the square	2.12: Completing the square	2.6	Students should be able to deal with expressions with a coefficient of x^2 other than 1. The most able students will appreciate that c is the greatest (or least) value of an expression of the form $a(x + b)^2 + c$	Work out the values of a , b and c such that $2x^2 + 6x + 7 \equiv a(x + b)^2 + c$	Completing the square
Keywords	Proportion, linear, quadratic, surd, exact, rational, irrational, rationalise, binomial, coefficient, completing the square, factorise	Career link	Chemical engineer https://www.unifrog.org/student/careers/school-subjects/chemical-engineer	Homework and assessment: Written tasks with past paper questions on topics studied.	

3. Algebra III – Functions and Graphs (7 hours)						
Topic	Specification link	Prior	Teaching points	Examples	Free resources	
Function notation	2.2: Definition of a function	2.6	Mapping diagrams can be useful when illustrating how a function works.	Notation $f(x)$ will be used, eg. $f(x) = x^2 - 9$	Functions	
Domain and range of a function	2.3: Domain and range of a function	2.2		Domain may be expressed as, for example, $x > 2$, or “for all x , except $x = 0$ ” and range may be expressed as $f(x) > -1$		
Composite functions	2.4: Composite functions	2.3		The result of two or more functions, say f and g , acting in succession. $fg(x)$ is g followed by f		
Graphs of functions	Graphs of linear functions	2.13: Drawing and sketching of functions. Interpretation of graphs. 3.1: Know and use the definition of a gradient 3.6: Draw a straight line from given information	2.14	Students must be comfortable sketching graphs on non-graph paper, indicating critical coordinates such as axis-intercepts.	Straight line graphs	
	Finding the equation of a line	3.5: The equation of a straight line $y = mx + c$ and $y - y_1 = m(x - x_1)$ and other forms	3.1	When finding the equation of a straight line, students are often reluctant to use the formula $y - y_1 = m(x - x_1)$ when they have successfully used $y = mx + c$ for a number of years.	Including interpretation of the gradient and y intercept from the equation	
	Graphs of quadratic functions	2.13: Drawing and sketching of functions. Interpretation of graphs.	2.12	Students will find it easier to get a smoothly curving and symmetric quadratic graph if the curve is drawn first, and the axes added afterwards.	Sketching quadratics	
	Inverse functions	2.5: Inverse functions	2.3	Use a number machine to illustrate a function and its inverse.	The inverse function of f is written f^{-1} Domains will be chosen for f to make f one-one	Inverse functions
	Graphs of exponential functions	2.13: Drawing and sketching of functions. Interpretation of graphs.	2.1	When sketching the exponential graph, students will find it easier to start near the asymptote and draw away from it.	Graphs could be linear, quadratic, exponential and restricted to no more than 3 domains. Exponential graphs will be of the form $y = ab^x$ and $y = ab^{-x}$ where a and b are rational numbers Sketch the graph of $y = x^2 - 5x + 6$ Label clearly any points of the intersection with the axes A function $f(x)$ is defined as	Exponential graphs
	Graphs of functions with up to three parts to their domains	2.13: Drawing and sketching of functions. Interpretation of graphs.	2.2 & Sketching linear graphs	Students may find it easier to first sketch all three graphs and then erase the parts not required.	$f(x) = \begin{cases} x^2 & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 3 - x^2 & 2 \leq x < 3 \end{cases}$ Draw the graph of $f(x)$ on the grid below for values of x from 0 to 3 Given a sketch of $y = ab^{-x}$, and two points, work out the values of a and b	3-part function
Keywords	Function, domain, range, composite, gradient, intercept, inverse, exponential, piecewise function	Career link	Computer programmer https://www.unifrog.org/student/careers/keywords/computer-programmer	Homework and assessment: Written tasks with past paper questions on topics studied.		

4. Algebra IV- Polynomials, Further Algebra and Sequences (9 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
Quadratic equations	2.14: Solution of quadratic equations	Linear equations	Students must be comfortable solving quadratic equations by factorising, completing the square and formula.		Quadratics
Simultaneous equations in two unknowns	2.15: Algebraic and graphical solution of simultaneous equations in two unknowns, where the equations could both be linear or one linear and one second order	2.14	The use of graphing software will help students to get a better understanding.	Solve $4x - 3y = 0$ and $6x + 15y = 13$ Solve $y = x + 2$ and $y^2 = 4x + 5$ Solve $y = x^2$ and $y - 5x = 6$ Solve $xy = 8$ and $x + y = 6$	Simultaneous equations
The factor theorem	2.11: Use of the factor theorem for rational values of the variable for polynomials	2.14	If a function has not been defined in a question, then students should be discouraged from using function notation without first defining their function, e.g. Let $f(x) = \dots$	Factorise $x^3 + x^2 - 5x + 6$ Show that $2x - 3$ is a factor of $2x^3 - x^2 - 7x + 6$ Solve $x^3 + x^2 - 10x + 8 = 0$ Show that $x - 7$ is a factor of $x^5 - 7x^4 - x + 7$	Factor+remainder theorem
Linear inequalities	2.17: Solution of linear inequalities	Linear equations	Students should be discouraged from replacing an inequality sign with an equals sign.	Solve $5(x - 7) > 2(x + 1)$	Linear inequalities
Quadratic inequalities	2.17: Solution of quadratic inequalities	2.14	A common error when solving inequalities is to fail to reverse the sign when multiplying or dividing by a negative value. Students should understand why the solution to $x(x - 3) > 0$ can be written as $x < 0$ or $x > 3$ but not $x < 0$ and $x > 3$	Solve $x^2 < 9$ Solve $2x^2 + 5x \leq 3$	Quadratic inequalities
Indices	2.18: Index laws, including fractional and negative indices and the solution of equations	2.14	Show/explore the patterns that lead to the conventional interpretation of negative and fractional indices.	Express as a single power of x $\sqrt{x^{\frac{1}{2}} \times x^{\frac{7}{2}}}$ $\sqrt{\frac{x^{\frac{3}{2}} \times x^{\frac{7}{2}}}{x^2}}$ Solve $x^{-\frac{1}{2}} = 3$ Solve $\sqrt{x} - \frac{10}{\sqrt{x}} = 3$ $x > 0$	Index laws
Algebraic proof	2.19: Algebraic proof	2.12 & 2.2	The presentation of a proof requires some 'work' from the spectator, particularly to interpret notation. Separate the argument from its presentation to make it more accessible for students.	Prove that $(n + 5)^2 - (n + 3)^2$ is divisible by 4 for any integer value of n	Algebraic proof
Sequences	2.20: Using n th terms of sequences 2.21: n th terms of linear sequences 2.22: n th terms of quadratic sequences	2.14	When finding the n th term of a sequence, students should be encouraged to check their answer by substituting values of n .	Work out the difference between the 16th and 6th terms of the sequence with n th term $\frac{2n}{n+4}$ A linear sequence starts 180 176 172 ... By using the n th term, work out which term has the value -1000 Work out the n th term of the linear sequence $r + s$ $r + 3s$ $r + 5s$... Work out the n th term of the quadratic sequence 10 16 18 16 ... Which term has the value 0?	Prime triangles
Limiting value of a sequence	2.20: Limiting value of a sequence as $n \rightarrow \infty$	Using n th terms of sequences	A spreadsheet may demonstrate the convergence more convincingly.	Write down the limiting value of $\frac{2n}{n+4}$ as $n \rightarrow \infty$	
Simultaneous equations in three unknowns	2.16: Algebraic solution of linear equations in three unknowns	2.15	Students should be encouraged to set out their work in a clear and systematic manner.	Solve $2x - 5y + 4z = 22$ $x + y + 2z = 4$ $x - y - 6z = -4$	3 unknowns
Keywords	Simultaneous equations, polynomial, factor theorem, inequality, index/indices, n th term, limiting value	Career link	Microbiologist https://www.unifrog.org/student/careers/school-subjects.k1/microbiologist	Homework and assessment: Written tasks with past paper questions on topics studied.	

5. Coordinate geometry (5 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
Parallel and perpendicular lines	3.2: Know the relationship between the gradients of parallel and perpendicular lines	3.1	It is essential to sketch diagrams in this topic. Add lines where appropriate; this often involves creating a right-angled triangle with shorter sides parallel to the axes. Pythagoras' theorem and gradients can then be used in solving the problem. Relate the formulae to their graphical representation to avoid errors in recall.	Show that A (0, 2), B (4, 6) and C (10, 0) form a right-angled triangle	Parallel & perp
The distance between two points	3.3: Use Pythagoras' theorem to calculate the distance between two points	GCSE		Distance2	
The midpoint of a line joining two points	3.4: Use ratio to find the coordinates of a point on a line given the coordinates of two other points	GCSE		midpoint	
Equation of a straight line	3.5: The equation of a straight line $y = mx + c$ and $y - y_1 = m(x - x_1)$ and other forms 3.2: Know the relationship between the gradients of parallel and perpendicular lines	3.1	When finding the equation of a straight line, students are often reluctant to use the formula $y - y_1 = m(x - x_1)$ when they have successfully used $y = mx + c$ for a number of years.		Point-slope equation
The intersection of two lines	2.15: Algebraic and graphical solution of simultaneous equations in two unknowns, where the equations could both be linear	GCSE	Graphing software can be useful here.		Desmos
Dividing a line in a given ratio	3.4: Use ratio to find the coordinates of a point on a line given the coordinates of two other points	Midpoint	Some students like to learn the formula whilst others prefer to consider each coordinate separately and apply some logic instead.		With a set-square
Equation of a circle	3.7: Understand that $x^2 + y^2 = r^2$ is the equation of a circle with centre (0, 0) and radius r	2.12	Students should practise converting between the two formats: $(x - a)^2 + (y - b)^2 = r^2$ and $x^2 + y^2 + fx + gy + h = 0$ Remembering the circle theorems learned at GCSE may help when solving circle problems.	Including writing down the equation of a circle given centre (0, 0) and radius The application of circle geometry facts where appropriate: <ul style="list-style-type: none"> ○ the angle in a semi-circle is 90°; ○ the perpendicular from the centre to a chord bisects the chord; ○ the angle between tangent and radius is 90°; ○ tangents from an external point are equal in length. 	Circles
	3.8: Understand that $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle with centre (a, b) and radius r			Including writing down the equation of any circle given centre and radius	
	3.9: The equation of a tangent at a point on a circle				
Keywords	Parallel, perpendicular, midpoint, chord, bisect, tangent, radius	Career link	Astronomer https://www.unifrog.org/student/careers/school-subjects.k1/astronomer	Homework and assessment: Written tasks with past paper questions on topics studied.	

6. Geometry I – Circle Theorems, Pythagoras and Trigonometry (5 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
Mensuration	6.1: Geometry – area & volume	GCSE	Students should already be familiar with these skills, but would benefit from a reminder.	Knowledge of perimeter and area of rectangles and circles; and of the area of triangles, parallelograms and trapezia; and of the surface area and volume of prisms, cylinders, spheres, cones and pyramids.	Volume & surface area
Pythagoras' theorem	6.4: Use of Pythagoras' theorem in 2D			Recognise Pythagorean triples; 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25 and simple multiples of these	Pythagoras
Angle facts	6.1: Geometry – angle properties			Knowledge of angle properties of parallel and intersecting lines, triangles, all special types of quadrilaterals and polygons.	Angle props
Circle theorems	6.1: Geometry – circle theorems			Understand and use circle theorems: <ul style="list-style-type: none"> ○ angle at the centre is twice the angle at the circumference; ○ angles in the same segment are equal; ○ opposite angles in cyclic quadrilateral add up to 180°; ○ alternate segment theorem; ○ the angle in a semi-circle is 90°; ○ the perpendicular from the centre to a chord bisects the chord; ○ the angle between tangent and radius is 90°; ○ tangents from an external point are equal in length. 	Circles
Geometric proof	6.2: Understand and construct geometrical proofs using formal arguments	6.1	The presentation of a proof requires some 'work' from the spectator, particularly to interpret notation. Separate the argument from its presentation to make it more accessible for students.	The use of theorems listed in the notes of section 6.1	Geometry proofs
Trigonometry in two dimensions	6.7: Be able to use the definitions $\sin \theta$, $\cos \theta$ and $\tan \theta$, for any positive angle up to 360° (measured in degrees only)	GCSE	Some students find the following to be useful: In the order 0°, 30°, 45°, 60°, 90°, the sin ratios are $\sqrt{\frac{0}{4}}$, $\sqrt{\frac{1}{4}}$, $\sqrt{\frac{2}{4}}$, $\sqrt{\frac{3}{4}}$, $\sqrt{\frac{4}{4}}$, with the cos ratios the same but in reverse, and the tan ratios being the quotient $\frac{\sin}{\cos}$	Angles measured anticlockwise will be taken as positive, angles measured clockwise will be taken as negative. (CAST diagram)	Trigonometry
	6.8: Knowledge and use of 30°, 60°, 90° triangles and 45°, 45°, 90° triangles			The use of the ratios $1 : \sqrt{3} : 2$ and $1 : 1 : \sqrt{2}$	
Trigonometric functions for angles of any size	6.6: Sketch and use graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size	6.7	It is worth spending time plotting the graph of $y = \sin x$ so that students have a deep sense of the symmetry of the graph.		
The sine and cosine graphs					
The tangent graph					
Solution of trigonometric equations	6.10: Solution of simple trigonometric equations in given intervals	6.6	Using knowledge of the shape of the graphs is the best way to work with the sin, cos and tan of any angle. When solving equations of the form $\sin 2x = \alpha$ a common error is to divide $\sin^{-1} \alpha$ by 2 before finding the other angles.	Equations will be restricted to single angles: $\sin x = 0.5$ $\sqrt{2} \sin x = \cos x$ for $0^\circ \leq x \leq 360^\circ$ $\sin^2 x = \frac{1}{4}$ for $0^\circ \leq x \leq 360^\circ$	
Trigonometric identities	6.9: Know and use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$	6.10	Students should understand the difference between $\sin^2 x$, $\sin x^2$, $\sin^{-1} x$, $(\sin x)^{-1}$ Use Pythagoras' theorem to demonstrate the identity $\sin^2 \theta + \cos^2 \theta = 1$	Including expressions to be simplified, proofs of identities and equations solved	

7. Geometry II- Further Trigonometry (4 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
The area of a triangle	6.3: Area of a triangle = $\frac{1}{2}ab \sin C$	6.7	Students will have met these at GCSE, but may be unsure of the ambiguous case of the sine rule.	Knowledge and use of trigonometry to solve right-angled triangles are expected	Triangle area
The sine rule	6.3: Sine and cosine rules in scalene triangles				Sine & cosine rules
The cosine rule					
Using the sine and cosine rules together					
Problems in three dimensions	6.4: Use of Pythagoras' theorem in 2D and 3D	6.4	Students tend to find the use of bearings in these questions particularly challenging. As always, a clear diagram is invaluable.	Including the angle between a line and a plane and the angle between two planes; including triangles that do not have right angles	3D trig
Lines and planes in three dimensions	6.5: Be able to apply trigonometry and Pythagoras' theorem to 2 and 3 dimensional problems	6.3 & 6.4			
Keywords	Segment, Polygon, Pythagorean triple, geometric proof, sine, cosine, tangent, trigonometric equation, trigonometric identities	Career link	Architect https://www.unifrog.org/student/careers/school-subjects.k1/architect	Homework and assessment: Written tasks with past paper questions on topics studied.	

Half Term 3: Calculus, Matrices

8. Calculus (6 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
The gradient of a curve	4.1: Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x	GCSE	Students should understand that $\frac{d}{dx}$ is the operator, and that y is the operand, when $\frac{dy}{dx}$ is written.	Including expressions which need to be simplified first Given $y = (3x + 2)(x - 3)$ work out $\frac{dy}{dx}$ Given $y = \frac{5}{x^3}$ work out $\frac{dy}{dx}$	Introducing calculus
Differentiation					
Differentiation using standard results	4.3: Differentiation of kx^n where n is an integer, and the sum of such functions	4.1			
Tangents and normal	4.2: Know that the gradient of a function is the gradient of the tangent at that point 4.4: The equation of a tangent and normal at any point on a curve	3.5 & 4.3	Students may find it useful to remember that a constant term added to a function simply translates its graph vertically and has no effect on its gradient. Hence a constant term differentiates to zero. A diagram can often be helpful when solving problems related to tangents and/or normal to curves.		
Increasing and decreasing functions	4.5: Increasing and decreasing functions	4.3		When the gradient is positive/negative a function is described as an increasing/decreasing function	Increasing & decreasing
The second derivative	4.6: Understand and use the notation $\frac{d^2y}{dx^2}$	4.5		Know that $\frac{d^2y}{dx^2}$ measures the rate of change of the gradient function	Maxima & minima
Stationary points	4.7: Use of differentiation to find maxima and minima points on a curve	4.6	The use of graph plotting software would help students to gain a better understanding of stationary points. Students should understand that a maximum or minimum is 'local' and is not necessarily a greatest or least value of the function.	Determine the nature either by using increasing and decreasing functions or $\frac{d^2y}{dx^2}$	
	4.8: Using calculus to find maxima and minima in simple problems			$V = 49x + \frac{81}{x} \quad x > 0$ Use calculus to show that V has a minimum value and work out the minimum value of V	
4.9: Sketch/interpret a curve with known maximum and minimum points					

Keywords	Differentiate, tangent, normal, increasing function, decreasing function, maxima, minima	Career link	Accountant https://www.unifrog.org/student/careers/keywords/management-accountant	Homework and assessment: Written tasks with past paper questions on topics studied.	Enrichment: Intermediate Mathematical Challenge (29th January 2025)
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9. Matrices (4 hours)					
Topic	Specification link	Prior	Teaching points	Examples	Free resources
Multiplying matrices	5.1: Multiplication of matrices	GCSE Maths	(All calculations will be restricted to 2×2 or 2×1 matrices) Students must appreciate the order in which matrices must be written when multiplying.	Multiplying a 2×2 matrix by a 2×2 matrix or by a 2×1 matrix	Matrices & linear transformations
Transformations				Multiplication by a scalar	
The identity matrix	5.2: The identity matrix I	5.1		2×2 only	
Transformations of the unit square	5.3: Transformations of the unit square in the x - y plane	5.2		Representation by a 2×2 matrix Transformations restricted to rotations of 90° , 180° or 270° about the origin, reflections in the lines $x = 0$, $y = 0$, $y = x$, $y = -x$ and enlargements centred on the origin	
Combining transformations	5.4: Combinations of transformations	5.3		Using matrix multiplications Use of i and j notation is not required	
Keywords	Matrix, identity matrix, scalar, transformation, unit square	Career link	Epidemiologist https://www.unifrog.org/student/careers/school-subjects/epidemiologist	Homework and assessment: Written tasks with past paper questions on topics studied.	