Year 12 Pure

Algebraic Expressions

Objectives. Students are expected to learn:

The laws of indices.

Negative and fractional indices.

Polynomials, expanding brackets and collecting like terms.

Factorising, including cubic expressions of the type $ax^3 + bx^2 + cx$.

Surds.

Surds: Rationalising the denominator.

Suggested time scale: 6 -7 lessons

Misconceptions

Students often leave surds un-simplified especially when rationalising a denominator using the complex conjugate.
Common errors include: misinterpreting $(a\sqrt{b})^2$ as $(a+\sqrt{b})^2$; evaluating $(\sqrt{2})^2$ as 4 instead of 2; slips when multiplying out brackets; basic arithmetic errors; and leaving surds in the denominator rather than fully simplifying fractions. Two examples of errors with indices are, writing $\frac{1}{3x}$ as 3x–1and writing $\frac{4}{\sqrt{x}}$ as $4x^{\frac{1}{2}}$; these have significant implications later in the course (e.g. differentiation).
Many of these errors can be avoided if students carefully check their work and have plenty of practice.
Students often struggle to evaluate indices involving both a fractional and negative power. This is especially true when the base involves an algebraic and numerical term.

Key Words Tier 2

Add, subtract, multiply, divide, fraction, simplify.

Key Words Tier 3

Expression, function, constant, variable, term, unknown, coefficient, index, linear, substitution, factorise, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, quadratic, quotient, intercepts.

Homework

Book 1 Mixed exercise Unit 1
Mathsgenie website: Algebraic Expressions

Career links

Actuarial Science, Biochemistry

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Quadratics

Objectives. Students are expected to learn:

Solving quadratic equations by factorising, completing the square and the formula.
(Candidates must be able to derive the formula.)

Quadratic functions and their graphs. The roots of a function, as well as their domain and range.

The use of the discriminant.

Modelling with quadratics: The use of quadratic functions to model real – life situations.

Suggested time scale: 6 -7 lessons

Misconceptions

Students often struggle manipulating quadratics when the x^2 term is negative.
Some students are unclear on the conditions for the number of roots when finding the discriminant.
Students often resort to using the formula when solving quadratics when it is more appropriate and efficient to use completing the square.
When sketching quadratic functions with no real roots students often make mistakes by drawing the curve below y = 0.

Key Words Tier 2

Modelling, solve, complete, derive, turning point, formula, draw, sketch.

Key Words Tier 3

Expression, function, constant, variable, term, unknown, coefficient, index, linear, identity, simultaneous, elimination, substitution, factorise, completing the square, intersection, change the subject, cross-multiply, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, sketch, plot, quadratic, maximum, minimum, turning point, transformation, translation, discriminant, real roots, repeated roots, intercepts, inequality.

Homework

Book 1 Unit 2 Mixed exercise
Mathsgenie website: Completing the Square, The Discriminant

Careers links

Aeronautical Engineering, Biomedical Sciences

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Equations and Inequalities

Objectives. Students are expected to learn:

Solving linear simultaneous equations

Solving quadratic simultaneous equations

Solving simultaneous equations on graph

Solving linear inequalities.

Solving quadratic inequalities

Representing inequalities on graphs (the use of dotted and solid lines required on graphs)

Suggested time scale: 7 - 8 lessons

Misconceptions

Students often forget to change the inequality sign when multiplying or dividing by a negative.
Students can get confused with the set notation for solving quadratic inequalities. Encourage them to sketch a graph and marked on the desired range of values. For instance, $x^2 - 7x + 12 < 0$ can have the incorrect solution of $x < 3$ and $x > 4$ rather than $3 < x < 4$.
When solving simultaneous equations students often forget to find both the x and y solutions after finding one.

Key Words Tier 2

Equation, equality, inequality, solve, linear, represent, turning point, below, above, rearrange, corresponding, sketch, coordinate, region.

Key Words Tier 3

Expression, function, constant, variable, term, unknown, coefficient, index, linear, identity, simultaneous, elimination, substitution, factorise, completing the square, intersection, change the subject, cross-multiply, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, sketch, plot, quadratic, maximum, minimum, turning point, transformation, translation, polynomial, discriminant, real roots, repeated roots, factor theorem, quotient, intercepts, inequality, asymptote .

Homework

Book 1 Unit 3 Mixed exercise

Mathsgenie website: Quadratics Inequalities and Simultaneous Equations

Career links

Chemical Engineering, Dentistry

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Enrichment

1) Senior Maths Challenge on 1st October 2024

We are regular participants in the maths challenge competition, and students speak highly of the chance to solve mathematical puzzles. This reinforces the foundations of mathematics as a problem-solving activity, and develops the skills required later by university admissions departments.

2) Free online Transition to University Course – Advanced Mathematics support programme Transition to University Course

Who is the course for?
The Transition to University Course 2024 course is designed for students who are:
Currently in Year 13 or taking a gap year.
Studying A level Mathematics.
Planning to begin studying a degree in maths, engineering, sciences, or closely related subjects such as economics in 2024 (or choosing to defer their entry until 2025).

Graphs and Transformations

Objectives. Students are expected to learn:

To sketch the graphs of simple functions, including:

- cubic graphs
- quartic graphs
- and reciprocal graphs $y = \frac{k}{x}$

Geometrical interpretation of algebraic solution of equations.

Use of intersection points of graphs of functions to solve equations.

Translating graphs: The effects of simple transformations on the graph of $y = f(x)$ as represented by $y = f(x)+k$ and $y = f(x+k)$
Stretching graphs: The effects of simple transformations on the graph of $y = f(x)$ as represented by $y = kf(x)$ and $y = f(kx)$
Reflecting graphs: The effects of simple transformations on the graph of $y = f(x)$ as represented by $y = -f(x)$ and $y = f(-x)$

Transforming graphs.
Candidates should be able to apply one of these transformations to quadratic, cubic or reciprocal graphs. Given the graph of any function y = f(x) candidates will be expected to sketch the graph resulting from one of the above transformations.

Suggested time scale: 7 - 8 lessons

Misconceptions

Students are often confused about the number of roots a polynomial has when they involve repeated roots.
When plotting cubic and quartic graphs, students often confuse the direction of curves.
Students lose examination marks by not labelling all the key coordinates where the curve passes through the axes.

Key Words Tier 2

Graphs, constant, coordinates, sketch, interpretation, solution, translating, stretching, reflecting, manipulate, effect, represent, transformation

Key Words Tier 3

Expression, function, constant, variable, term, unknown, coefficient, index, linear, identity, simultaneous, elimination, substitution, factorise, completing the square, intersection, change the subject, cross-multiply, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, sketch, plot, quadratic, maximum, minimum, turning point, transformation, translation, polynomial, discriminant, real roots, repeated roots, factor theorem, quotient, intercepts, inequality, asymptote .

Homework

Book 1 Unit 4 Mixed exercise
Mathsgenie website: Sketching and Transforming Curves

Careers links

Civil Engineering, Chemistry

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Algebra and Functions

Straight Line Graphs

Objectives. Students are expected to learn:

The equation of a straight line, including the forms:
$y = mx + c$ and $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.

The equation of a line through two given points.

The condition for two straight lines to be parallel or perpendicular, including equations of lines parallel or perpendicular to a given line through a given point.

The distance between two given points: d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Modelling with straight lines: to represent a real – life situation using mathematical concepts.

Suggested time scale: 5 - 6 lessons

Misconceptions

Students can often become confused with the algebraic workings due to not drawing diagrams or diagrams lacking sufficient detail.
A common mistake is to write the correct gradient of line in the form y = mx + c but write it incorrectly when converting the equation into the form ax + by + c = 0.

Key Words Tier 2

Line, formula, straight, point, coordinate, condition, parallel, perpendicular, distance, represent, situation, concept, modelling, product, formula, origin, assumption, create.

Key Words Tier 3

Equation, bisect, centre, chord, circle, circumcircle, coefficient, constant, diameter, gradient, hypotenuse, intercept, isosceles, linear, midpoint, parallel, perpendicular, proportion, Pythagoras, radius, right angle, segment, semicircle, simultaneous, tangent.

Homework

Book 1 Unit 5 Mixed exercise
Mathsgenie website: The Equation of a Line

Careers links

Economics, Environmental Science/Studies

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Circles

Objectives. Students are expected to learn:

Assumed GCSE knowledge to specifically include:
The angle in a semi circle is a right angle.
The perpendicular from the centre to a chord bisects the chord.
The perpendicularity of radius and tangent.

To find midpoint of a line segment and perpendicular bisectors.

The equation of a circle in the form $(x-a)^2+(y-b)^2=r^2$, and $x^2+gx+y^2+hx+k=0$.
Candidates should be able to find the co-ordinates of the centre of a circle and it's radius given the equation of a circle.

To find intersections of straight lines and circles.

To use tangent and chord properties.

To solve problems including circles and triangles.

Suggested time scale: 6 - 7 lessons

Misconceptions

The coordinates of the centre of a circle are sometimes given with the negatives.
For instance, $(x-3)^2+(y-4)^2=r^2$ can have the incorrect centre as (-3, -4).
Students can often become confused with the algebraic workings due to not drawing diagrams or diagrams lacking sufficient detail.

Key Words Tier 2

Circle, angle, intersect, solve, coordinate.

Key Words Tier 3

Equation, bisect, centre, chord, circle, circumcircle, coefficient, constant, diameter, gradient, hypotenuse, intercept, isosceles, linear, midpoint, parallel, perpendicular, proportion, Pythagoras, radius, right angle, segment, semicircle, simultaneous, tangent.

Homework

Book 1 Unit 6 Mixed exercise
Mathsgenie website: The Equation of a Circle

Careers links

Electrical/Electronic Engineering, Geology/Earth Sciences

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Coordinate geometry

Algebraic Methods

Objectives. Students are expected to learn:

To simplify algebraic fractions by factorising the numerator and/or the denominator

The division of a polynomial by $(x \pm a)$.

The Factor theorem for division of f(x) by $(x \pm a)$ and $(ax \pm b)$.

Factorising cubic expressions of the form $ax^3 + bx^2 + cx + d$.

Methods of Proof:

- Proof by Deduction
- Proof by Exhaustion
- Disproof by Counter Example

Suggested time scale: 5 - 6 lessons

Misconceptions

Students often forget to give a written conclusion as the final part of their proof.
When solving cubic equations, mistakes are sometimes made when substituting in negative values of *x*, particularly with the cubic term.
Some students try to use the long division method to factorise quadratics which can be more easily solved by factorisation.
Students often lose marks when asked to prove whether an expression is divisible by or a multiple of a constant. Encourage them to consider the variable as an odd and an even value.
Students often lose marks by not concluding that a squared term can never be negative.

Key Words Tier 2

Simplify, prove, proof, disproof, deduction, manipulate, counter-example, exhaustion, assumptions, conclusion, therefore, conjecture, prediction, implies, converse, sufficient.

Key Words Tier 3

Polynomials, factorisation, quadratic, cubic, quartic, rational number, fully factorise, factor, expand.

Homework

Book 1 Unit 7 Mixed exercise
Mathsgenie website: The Factor Theorem and Algebraic Division, Proof

Careers links

Engineering (General), Engineering (General)

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

The binomial expansion

Objectives. Students are expected to learn:

The use of Pascal's triangle

The use of the notations $n!$ and $\binom{n}{r}$.

The expansion of $(1+x)^n$, for positive integer n, using the binomial series.

To solve binomial problems.

To use the binomial expansion to find simple approximations for complicated functions.

Suggested time scale: 5 - 6 lessons

Misconceptions

Students often forget that the power rule applies when raising terms such as $(ax)^n$.
For example, students incorrectly write this as ax^n instead of $a^n x^n$.
Often, students forget to convert expressions in the form $(a + x)^n$ to $(1 + x)^n$.

Key Words Tier 2

Adjacent, expansion, notation, general, approximate.

Key Words Tier 3

Binomial, coefficient, probability, factorial, factorisation, quadratic, cubic, quartic, fully factorise, factor, expand.

Homework

Book 1 Unit 8 Mixed exercise
Mathsgenie website: The Binomial Expansion

Careers links

Mathematics, Medicine

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Further Algebra (Binomial, Factor Theorem)

Trigonometric Ratios

Objectives. Students are expected to learn:

The use of the Sine and the Cosine Rules.

The area of a triangle in the form $\frac{1}{2}ab\sin C$.

To solve triangle problems.

To sketch the graphs of sine, cosine and tangent functions

To review basic trig graphs, their graphs symmetries and periodicity.
Candidates will be expected to have knowledge of the graphs of the form $y = a\sin(bx + c)$.

Suggested time scale: 6 - 7 lessons

Misconceptions

Students often mislabel their diagrams, so the angle and opposite edge do not have the respective upper- and lower-case letters.

Problems involving an angle to be found using the Sine Rule can have two solutions.
When transforming graphs, students should use sketched diagrams are mistakes are often made when working algebraically.

Key Words Tier 2

Interval, periodic, amplitude, inverse, degree, identity, symmetry, opposite, adjacent.

Key Words Tier 3

Sine, cosine, tangent, function, angle of elevation, angle of depression, bearing, special angles, unit circle, hypotenuse, intercept.

Homework

Book 1 Unit 9 Mixed exercise
Mathsgenie website: Sine Rule, Cosine Rule, Area of Any Triangle

Careers links

Mechanical Engineering, Optometry

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Trigonometric Identities and Equations

Objectives. Students are expected to learn:

The use of the CAST diagram or angles in all four quadrants.

The exact trigonometrical values for common angles.

To know and use the trigonometric identities of $\tan\theta = \frac{\sin\theta}{\cos\theta}$. and $\sin^2\theta + \cos^2\theta = 1$.

To solve simple trigonometrical equations

To solve more complex trigonometrical equations of the form

$\sin(x + 30) = 0.5$ for $-180 \le \theta \le 180$

$\cos 2x = \frac{\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$

To solve trigonometrical equations by using trigonometric identities

Suggested time scale: 6 - 7 lessons

Misconceptions

When solving trigonometric equations students lose marks in the following ways:

- not finding all the solutions within the correct range
- including solutions that fall outside of the given range
- giving solutions for the transformed equation rather than x. For instance, giving solutions for 2x or x + 30° rather than x.

Key Words Tier 2

Interval, periodic, amplitude, inverse, degree, identity, symmetry, opposite, adjacent.

Key Words Tier 3

Sine, cosine, tangent, function, special angles, unit circle, hypotenuse, principal value, acute, ratio.

Homework

Book 1 Unit 10 Mixed exercise
Mathsgenie website: Solving Trigonometric Equations

Careers links

Physiotherapy, Statistics

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Trigonometry

Vectors

Objectives. Students are expected to learn:

The representation of vectors in 2 dimensions.
The magnitude and direction of a vector.
Unit vectors

The addition of vectors. The multiplication of a vector by a scalar.
Position vectors, including the distance between two points.
Solving geometrical problems with vectors

- Ratio Theorem: to divide a line segment with a point
- Comparing coefficients

Use vectors to solve problems in pure mathematics and in context (speed, velocity, etc.)

Suggested time scale: 6 - 7 lessons

Misconceptions

Some students lose marks by writing the magnitude of a vector as a negative.
When writing vectors in i and j notation, some students incorrectly think of it as a coordinate pair and write 3i – 2j as (3i, -2j).
Some students get confused knowing when a vector gives a direction or a position.

Key Words Tier 2

Direction, component, dimension, opposite, respectively.

Key Words Tier 3

Vector, scalar, magnitude, parallel, perpendicular, modulus, ratio, collinear, scalar product, position vectors.

Homework

Book 1 Unit 11 Mixed exercise
Mathsgenie website: Vectors

Careers links

Physics, Pharmacy

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Vectors

Differentiation

Objectives. Students are expected to learn:

Introduction to Calculus: In this unit the notations y or f(x) may be used and the notations $\frac{dy}{dx}$ or f'(x).

The derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point.
The gradient of the tangent as a limit.

Interpretation as a rate of change, i.e knowledge that $\frac{dy}{dx}$ is the rate of change of y with respect to x.

Differentiation from first principles for small positive integer powers and $sinx$ and $cosx$
Eg students should be able to use n = 2 and n = 3, the gradient expression $\lim_{h\to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$
Students may use δx or h

The differentiation of x^n and related sums and differences. To include expressions like $(2x+1)(3x-4)$ or $\frac{x^2+3x-6}{4x^{\frac{1}{2}}}$

Application of Differentiation to gradients of tangents and normals.

Increasing and decreasing functions.

Second order derivatives. Stationary points.

Sketching the gradient function for a given curve.

Modelling with differentiation, rate of change.

Suggested time scale: 12 - 13 lessons

Misconceptions

A small percentage of students confuse differentiation with integration when answering exam questions.
Some students use the second derivative to determine whether a function is increasing or decreasing when they should use $f'(x) \geq 0$ or $f'(x) \leq 0$.
When asked to find the gradient of a tangent to a point on a curve, some students incorrectly make the gradient of the curve equal to zero and attempt to find x.
Students can struggle knowing the conditions for maxima and minima turning points.
Some students lose marks in their differentiation by not dropping the constant in the original function and not simplifying surds.
When asked to find maximum and minimum turning points, some students substituted a value of x on either side of f'(x)=0, which requires more work than using the second derivatives.
Exam questions often link applying differentiation to volume and surface area. As a result, some students lose marks deriving the volume or surface area equations, leading to incorrect derivatives.

Key Words Tier 2

Rational, increasing, decreasing, maximum, minimum.

Key Words Tier 3

Differentiation, derivative, first principles, rate of change, constant, tangent, normal, stationary point, point of inflection, integer, calculus, function, parallel, perpendicular.

Homework

Book 1 Unit 12 Mixed exercise
Mathsgenie website: Differentiation from First Principles, Differentiation

Careers links

Accountancy, Psychology

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Differentiation

Exponentials and Logarithms

Objectives. Students are expected to learn:

Exponential Functions: The curve y = a^x and y = e^x and their basic transformations.

Exponential modelling: rate of increase/decrease

The inverse of exponential functions are called logarithms. $\log_a n = x$ is equivalent to $a^x = n$ ($a \neq 1$)

The laws of logarithms, to include the following

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$k \log_a x = \log_a \left(x^k\right)$

$\log_a \left(\frac{1}{x}\right) = -\log_a x$

$\log_a a = 1$

Solving equations using logarithms

Working with natural logarithms. $\ln x = \log_e x$

Logarithms and non-linear data
Use of Logarithmic graphs to estimate parameters in relationships of the form
$y = ax^n$ and $y = kb^x$ given data for x and y
Plot $log\ y$ against $log\ x$ and obtain a straight line where the intercept is $log\ a$ and the gradient is n.
Plot $log\ y$ against x and obtain a straight line where the intercept is $log\ k$ and the gradient is $log\ b$

Suggested time scale: 9 - 10 lessons

Misconceptions

Errors seen in exam questions where students have to sketch exponential curves include: stopping the curve at x = 0; getting the wrong y-intercept; and believing the curve levels off to y = 1 for x < 0.
When using laws of logs to answer proof or 'show that' questions, students must show all the steps clearly and not have jumps in their working out.

Key Words Tier 2

Equivalent, reflection, recognise, solve.

Key Words Tier 3

Exponential, exponent, power, logarithm, base, initial, rate of change, compound interest

Homework

Book 1 Unit 14 Mixed exercise

Mathsgenie website: Exponentials and Logarithms

Careers links

Computer Science, Sports Science/Physical Education

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Exponentials and Logarithms

Integration

Objectives. Students are expected to learn:

Indefinite Integration as the reverse of differentiation. Finding functions: to include the constant of integration.

The integration of x^n. To include expressions like $\frac{3}{4}x^5 - 4\sqrt{x}$ or $\frac{(x+2)^2}{x^{\frac{1}{2}}}$

Given f'(x) and a point on a curve, candidates should be able to find an equation of the curve in the form y = f(x).

Definite integrals: to calculate an integral between two limits.

Areas under curves by using definite integration

Areas under the *x* – axis: when the area is bounded by a curve and the *x* – axis is below the *x* – axis the integral returns a negative answer.

Areas between curves and lines. The use of definite integrals together with areas of trapeziums and triangles to find more complicated areas on graphs.

Suggested time scale: 8 - 9 lessons

Misconceptions

Students sometimes have difficulty when integrating expressions involving negative indices. Forgetting to add + c when working out indefinite integrals is also a very common mistake.
Lack of algebraic fluency can cause problems for some students, particularly when negative/fractional indices are involved or when a negative number is raised to a power. Arithmetic slips are also a common cause of lost marks, often when negative numbers are substituted and subtracted after integration.
Students are generally more successful if they expand any brackets before attempting to integrate the function.

Key Words Tier 2

Apply, separately, value, solve, area.

Key Words Tier 3

Calculus, differentiate, integrate, reverse, indefinite, definite, constant, evaluate, intersection.

Homework

Book 1 Unit 13 Mixed exercise

Mathsgenie website: Integration

Careers links

Architecture, Management Studies

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Integration

Proof and Partial Fractions

Objectives. Students are expected to learn:

Proof by contradiction

Simplifying algebraic fractions: add, subtract, multiply and divide algebraic fractions

Partial Fractions:

- to include denominators with 2 or 3 distinct linear factors
- repeated linear factors
- problems where the degree of the numerator is equal to or exceeds the degree of the denominator (improper fractions with the use of algebraic division)

Suggested time scale: 5 - 6 lessons

Misconceptions

Some students mistakenly think that substituting several values into an expression is sufficient to prove the statement for all values.
Similarly, for example, referring to a graph to prove that the gradient is always positive rather than completing the square will not gain marks for a proof.
Students need to practise factorising quadratics as this is often done incorrectly.
The most common errors include failing to include all necessary brackets, casual miswriting of signs part way through calculations and not dealing correctly with factors. Particular care with signs needs to be taken when a fraction follows a minus sign.
Some students will set up and solve simultaneous equations rather than using values of x to work out missing constants.
Ensure students are aware of the most efficient methods for solving different types of problem so they do not waste time in exam situations.

Key Words Tier 2

Proof, verify, deduction, contradict, assumption, contradiction.

Key Words Tier 3

Rational, irrational, square, root, prime, infinity, square number, quadratic, expansion, trigonometry, Pythagoras, polynomial, numerator, denominator, factor, difference of two squares, quadratic, power, index, coefficient, degree, squared, coefficients, improper, identity, algebraic fraction, partial fraction.

Homework

Book 2 Unit 1 Mixed exercise
Mathsgenie website: Proof by Contradiction, Partial Fractions

Careers links

Investment banking, Meteorology

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessments: Proof Algebraic and partial fractions

Sequences and Series

Objectives. Students are expected to learn:

Arithmetic sequences (progressions). Common difference, n^{th} term or general term.

Arithmetic Series: the sum of the terms of an arithmetic sequence. Candidates need to be able to prove this formula.

Geometric sequences. Common ratio, n^{th} term or general term.

Geometric Series: the sum of the terms of an geometric sequence. Candidates need to be able to prove this formula.

The sum to infinity of a convergent geometric series ($-1 < r < 1$).

Sigma notation to signify a sum ($\sum$).

Recurrence relations: Sequences generated by a formula for the n^{th} term or a simple relation of the form $x_{n+1} = f(x_n)$.
Encourage the use of the ANS button on a calculator to obtain the terms for a recurrence relation.

To include the terminology increasing, decreasing, periodic sequences .

Modelling with series: real – life situations using mathematical concepts.

Suggested time scale: 8 - 9 lessons

Misconceptions

When working with formulae for sequences and series, it is important that students state the relevant formula before substituting so that method marks can be awarded even if there is a numerical slip.

A fairly common error is to mix up the formulae for sums and terms, for example finding Sn rather than Un and vice-versa.

When asked to find the limit of u_n some candidates use the sum to infinity of a geometric series

Key Words Tier 2

Consecutive, limit, order, sum, difference.

Key Words Tier 3

Sequence, series, finite, infinite, summation notation, Σ(sigma), periodicity, convergent, divergent, natural numbers, arithmetic series, arithmetic progression (AP), common difference, geometric series, geometric progression (GP), common ratio, nth term, sum to n terms, sum to infinity (S_∞).

Homework

Book 2 Unit 3 Mixed exercise
Mathsgenie website: Recurrence Relations, Arithmetic Sequences and Series, Geometric Sequences and Series

Careers links

Economics, computer programming

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Sequences and Series

Year 13 Pure

Functions and Graphs

Objectives. Students are expected to learn:

Definition of a function : a function may be a one-one mapping or a many-one mapping : the notation $f: x \to$ and $f(x)$ will be used.
Domain and range of functions.
Composite functions: fg(x) means apply g first, then apply f; fg(x) = f(g(x))

The modulus function and their graphs, to include $y = |f(x)|$ and $y = |ax + b|$

The transformation from $y = f(x)$ to $y = |f(x)|$ and $y = f|x|$.
Knowledge of the effect of simple transformations on the graph $y = f(x)$ as represented by $y = af(x)$, $y = f(ax)$, $y = f(x) + a$, $y = f(x + a)$, $y = -f(x)$, $y = f(-x)$.

Inverse functions, and their graphs, $y = f^{-1}(x)$. The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of each other in the line $y = x$.

Solving modulus problems

Suggested time scale: 7 - 8 lessons

Misconceptions

Students may find it difficult to sketch graphs involving modulus functions particularly if they are combined with other functions, for example logarithms.
In exam situations, often only the highest scoring students are able to solve modulus equations with x on both sides, or inequalities which involve the modulus function.
Students can often successfully find the range in exam questions, but some give their answer in terms of x rather than f(x).
When finding inverse functions, students need to remember to swap x and y. When describing why a function does not have an inverse, students should be advised to answer this question as “because it is not one to one” or “because it is many to one”.
Students often score well on questions which involve describing geometrical transformations, but incorrect use of terminology will lose marks. Students must use the correct terms: stretch, scale factor and translation.
Students also need to be aware that the order of transformations is often important.

Key Words Tier 2

Translate, inverse, stretch, reflect, composite, inverse, transformation.

Key Words Tier 3

Function, mapping, domain, range, modulus, one to one, many to one, mappings, f(x), fg(x), $f^{-1}(x)$.

Homework

Book 2 Unit 2 Mixed exercise
Mathsgenie website: Functions, Transforming Graphs

Careers links

Veterinary Medicine, Research Scientist

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Functions and Modelling

Enrichment

1) Senior Maths Challenge on 1st October 2024

We are regular participants in the maths challenge competition, and students speak highly of the chance to solve mathematical puzzles. This reinforces the foundations of mathematics as a problem-solving activity, and develops the skills required later by university admissions departments.

2) Free online Transition to University Course – Advanced Mathematics support programme Transition to University Course

Who is the course for?
The Transition to University Course 2024 course is designed for students who are:
Currently in Year 13 or taking a gap year.
Studying A level Mathematics.
Planning to begin studying a degree in maths, engineering, sciences, or closely related subjects such as economics in 2024 (or choosing to defer their entry until 2025).

Binomial Expansion

Objectives. Students are expected to learn:

The Binomial series for rational n. Expanding $(1 + x)^n$ and $(a + bx)^n$.
Be aware expansion is valid for $\left|\frac{bx}{a}\right| < 1$

To include the expansion of rational functions by decomposition into partial fractions.

To use the binomial expansion to find simple approximations for complicated functions.

Suggested time scale: 3 - 4 lessons

Misconceptions

When expanding $(1 + 4x)^{\frac{1}{2}}$ most students got the first two terms of the expansion correct, but often there was a mistake in the x2 term, with 4x becoming just x being the common error. Some students made arithmetic errors with 42, by failing to actually square the 4, and others failed to simplify the binomial coefficient correctly.
When expanding an expression of the form (a + x)n a common error is to write this as a$(1 + \frac{x}{a})$n rather than an$(1 + \frac{x}{a})$n.
Other errors include algebraic errors when combining two expansions, doing more work than is necessary when, for example, only terms up to x2 are required, including the equality in the expression for the range of valid values for x and lack of understanding when using the modulus symbol (writing expressions such as $|x| < -4$).

Key Words Tier 2

Expansion, theorem, approximation, substitution.

Key Words Tier 3

Binomial, integer, rational, power, index, coefficient, validity, modulus, factorial, ${}^{n}Cr$, combinations, Pascal's triangle, partial fractions, approximation, converges, diverges, root.

Homework

Book 2 Unit 4 Mixed exercise
Mathsgenie website: The Binomial Expansion

Careers links

Data analyst, Law

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: The Binomial Theorem

Radians

Objectives. Students are expected to learn:

Radian measure. $2\pi = 360^\circ$, $\pi = 180^\circ$ and 1 radian = $\frac{180^\circ}{\pi}$.

Arc length using the formula $s = r\theta$

Area of sector and segments using the formulae $A = \frac{1}{2}r^2\theta$ and $A = \frac{1}{2}r^2(\theta - sin\theta)$ respectively.

Solving trigonometric equations in radians

Small angles approximations: $sin\theta \approx \theta$, $tan\theta \approx \theta$ and $cos\theta \approx 1 - \frac{\theta^2}{2}$.

Suggested time scale: 5 - 6 lessons

Misconceptions

A common exam mistake is for students to have their calculators set in the wrong mode resulting in the loss of accuracy marks.
Students may try to use these approximations when angles are measured in degrees rather than radians.

Key Words Tier 2

Opposite, exact, symmetry, contain, measure, infinity, identity, proof, approximation, interval, infinity, adjacent, sector, segment, area, solve.

Key Words Tier 3

Pythagoras, Pythagorean triple, right-angled triangle, opposite, hypotenuse, trigonometry, sine, cosine, tangent, secant, cosecant, cotangent, SOHCAHTOA, exact, symmetry, periodicity, equation, quadrant, degree, radian, circular measure, asymptote, small angles.

Homework

Book 2 Unit 5 Mixed exercise
Mathsgenie website: Radians, Small Angle Approximations

Careers links

Product designer, Finance (Fraud investigation)

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Trigonometric Functions

Objectives. Students are expected to learn:

Knowledge of secant, cosecant and cotangent and their graphs ($y = \sec x$, $y = \operatorname{cosec} x$ and $y = \cot x$); as well as their domains and ranges.
Using sec *x*, cosec *x* and cot *x* to simplify expressions, prove identities and solve equations.

Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.

Inverse trigonometric functions: Knowledge and use of arcsin, arccos and arctan, their graphs, domains and ranges.

Suggested time scale: 5 - 6 lessons

Misconceptions

The most common errors in these questions involve using wrong notation, for example sin x2 instead of sin2 x, or making algebraic mistakes. Students sometimes struggle to deal with more complicated functions such as cosec (3x + 1) and do not always recognise where trigonometric identities can be used.

Key Words Tier 2

Opposite, exact, symmetry, contain, measure, infinity, identity, proof, approximation, interval, infinity, adjacent, sector, segment.

Key Words Tier 3

Pythagoras, Pythagorean triple, right-angled triangle, opposite, hypotenuse, trigonometry, sine, cosine, tangent, secant, cosecant, cotangent, SOHCAHTOA, exact, symmetry, periodicity, equation, quadrant, degree, radian, circular measure, asymptote, small angles.

Homework

Book 2 Unit 6 Mixed exercise
Mathsgenie website: Sec, Cosec and Cot, Trig identities

Careers links

Air traffic controller, Criminologist

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Trigonometry and modelling

Objectives. Students are expected to learn:

Knowledge and use of the Addition formulae (Compound angle formulae).

Knowledge and use of Double angle formulae

Solving trigonometric equations using the addition and double-angle formulae.

Simplifying $a \cos x \pm b \sin x$ (The $R\cos(\theta \pm \alpha)$ method).

Proving trigonometric identities. **It is essential that students know which formulae are provided in the formulae book and which have to be learnt.**

Using trigonometric functions to model real-life situations.

Suggested time scale: 7 - 8 lessons

Misconceptions

The most common errors are sign errors when using the compound and double angle formulae.
When writing $a\cos\theta + b\sin\theta$ into the form $R\sin(\theta - \alpha)$ most students found the value of R correctly, the same was not true of the angle α. Some students seemingly failed to notice that α was given as an acute angle.
When solving an equation of the form $a\cos\theta + b\sin\theta = c$ many students seemingly could not cope with the result of $-39.23°$ that their calculator gave them and could not get the first solution. In addition some students found the third quadrant solution only, whereas some found more than two solutions. However many students did give a fully correct solution, often by using a sketch graph to help them decide where the solutions lay.
These questions often prove to be the most demanding on the paper and serve to differentiate between students.
Students need to make sure they include all steps in the proof with full explanation.

Key Words Tier 2

Addition, compound, double, formulae, express.

Key Words Tier 3

Pythagoras, Pythagorean triple, right-angled triangle, opposite, adjacent, hypotenuse, trigonometry, sine, cosine, tangent, secant, cosecant, cotangent, SOHCAHTOA, exact, symmetry, periodicity, identity, equation, interval, quadrant, degree, radian, circular measure, infinity, asymptote, small angles, approximation, identity, proof.

Homework

Book 2 Unit 7 Mixed exercise
Mathsgenie website: Addition and Double Angle Formulae, R Formulae

Careers links

Cyber intelligence officer, Insurance underwriter

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Trigonometry

Parametric Equations

Objectives. Students are expected to learn:

Candidates will be expected to sketch a curve from its parametric form.

Parametric equations of curves, to include the conversion between Cartesian and Parametric forms.

Using trigonometric identities with parametric equations.

Points of intersections with parametric equations.

Using parametric equations to model real-life situations.

Suggested time scale: 5 - 6 lessons

Misconceptions

Students may have difficulties making any progress with these sorts of questions if they cannot work out which trigonometric identity to apply when eliminating the parameter t.

Key Words Tier 2

Convert, parameter, identity, eliminate, substitute, modelling.

Key Words Tier 3

Parametric, Cartesian, circle, hyperbola, parabola, ellipse, domain.

Homework

Book 2 Unit 8 Mixed exercise
Mathsgenie website: Parametric Equations

Careers links

Software developer, Stockbroker

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Parametric Equations

Further Differentiation

Objectives. Students are expected to learn:

Differentiation from first principles for $sinx$ and $cosx$.

Differentiating e^x, a^x and ln$x$.

The chain rule.
The product rule.
The quotient rule

Differentiation of trig functions, to include sin x, cos x, tan x, sec x, cosec x and cot x.

Differentiation of inverse trig functions $arcsinx, arcosx$ and $arctanx$

Parametric differentiation, to include the equations of tangents and normals.

Implicit differentiation
Using second derivatives (concave and convex functions, point of inflections).

Connected rates of change, writing differential equations.

Suggested time scale: 12 - 14 lessons

Misconceptions

Students often miss out minus signs or add an extra x into the answer when differentiating expressions like $e^{-\frac{1}{4}x}$.

Some students mix up $\frac{dx}{dy}$ and $\frac{dy}{dx}$ and others struggle to differentiate functions involving ln. For example given when differentiating y = ln 6x they write $\frac{1}{6x}$ rather than $\frac{1}{x}$.

Common errors involve: not using the method specified; algebraic errors when manipulating expressions; and being unable to identify the need of the product rule and instead simply differentiating the separate parts and multiplying.

Students should be encouraged to state "$\frac{dx}{dy}$ = … when x = …", especially when finding a given answer.

An easy mistake students may make is to mix up maxima and minima.

Most students are able to substitute correctly into a formula for exponential growth and decay.

When required to set up an inequality most students showed that they understood the information given and wrote down a correct opening expression, although there was uncertainty over which way the inequality should go. Some then simplified and solved using logarithms efficiently to get the correct answer. Some resorted to trial and improvement which was accepted for full marks if done correctly, but was worth no marks otherwise.
When solving equations involving exponentials, knowledge of using logarithms varied widely. Many were unable to deal properly with the coefficient and the exponential term and wrote down equations in which t actually should have cancelled out.

Some care needs to be taken when interpreting the answers to exponential growth and decay questions to ensure they are given in the correct form e.g. to the nearest year, second etc.

Key Words Tier 2

Turning point, maximum, minimum, implicit, product.

Key Words Tier 3

Derivative, tangent, normal, turning point, stationary point, inflexion, parametric, differential equation, rate of change, quotient, first derivative, second derivative, increasing function, decreasing function.

Homework

Book 2 Unit 9 Mixed exercise
Mathsgenie website: The Chain Rule, The Product Rule, The Quotient Rule, Trigonometric Differentiation, Implicit Differentiation, Cos and Sin from First Principles

Careers links

Tax adviser, Acoustics consultant

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Further Differentiation

Numerical Methods

Objectives. Students are expected to learn:

Locating roots of $f(x) = 0$ by considering changes of sign of $f(x)$.

Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.

Solve equations approximately using simple iterative methods: be able to draw associated cobweb and staircase diagrams.

Solve equations using Newton-Raphson method and other recurrence relations of the form $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Use numerical methods to solve problems in context.

Suggested time scale: 4 - 5 lessons

Misconceptions

Students must define f(x) before substituting x-values to find a root.
Most students can successfully identify the root of equations. However there are still many students who then write "change of sign therefore a root" without clarification of where the root lies and hence loose a mark.
Marks are sometimes lost unnecessarily if students do not give their answers to the specified number of significant figures or decimal places.
Marks will be lost due to using degrees (instead of radians) if functions involve trigonometric terms.
Choosing an unsuitable interval will also prevent progress in these questions.
Marks are often lost for sign errors and other numerical slips.
Students must show full working leading to the correct answer for full marks. Giving a correct answer either without working or following wrong working will result in zero marks.

Key Words Tier 2

Interval, satisfy, method, rearrange, approximating.

Key Words Tier 3

Roots, continuous, function, positive, negative, converge, diverge, derivative, tangent, chord, iteration, Newton-Raphson, staircase, cobweb.

Homework

Book 2 Unit 10 Mixed exercise
Mathsgenie website: Iteration, Newton-Raphson, The Trapezium Rule

Careers links

Sound engineer, Game designer

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Numerical Methods

Further Vectors

Objectives. Students are expected to learn:

3D coordinates and the representation of vectors in 3 dimensions.

The magnitude and direction of a 3D vector.

Position vectors, including the distance between two points using 3D vectors.

Finding angles between 3D vectors and the positive coordinate axes.

Solving geometrical problems with 3D vectors (comparing coefficients).

Application to mechanics

Suggested time scale: 4 - 5 lessons

Misconceptions

Encourage students to draw diagrams to help their geometrical thinking when answering vector questions.
Stress the importance of reading the question carefully and giving answers in the correct way, for example coordinates or column vectors may be requested.
Emphasise the importance of good notation. Students do not always understand that AP^2 represents the square of the length AP.

Key Words Tier 2

Origin, distance, direction, coefficient, plane, compare.

Key Words Tier 3

Vector, scalar, column, 3D coordinates, vertices, Cartesian, i, j, k, magnitude, angle, position vector, unit vector, orthogonal, vector addition/subtraction.

Homework

Book 2 Unit 11 Mixed exercise
Mathsgenie website: 3D Vectors

Careers links

Quantity surveyor, Radiation protection practitioner

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: Further Vectors

Further Integration

Objectives. Students are expected to learn:

Integration of standard functions including x^n , e^x, $\frac{1}{x}$, sin x, cos x, $\sec^2 x$, cosec$x$cot$x$, $\text{cosec}^2 x$, sec$x$tan$x$ and associated functions.

Integrating functions in the form of $f(ax + b)$.

Integration of rational fractions to include problems leading to natural logarithms. (Reverse chain rule).
Integration of linear brackets. (Reverse chain rule).

Integration using trigonometric identities. Standard results for $\sin^2 x$ etc.

Substitution includes finding a suitable substitution.

Integration by parts, including the integration of ln x.

Integration by parts, including more than one application.

Integration of algebraic fractions with the use of partial fractions.

Finding the area under a curve given both in cartesian and parametric form.

The trapezium rule.

The general and particular solution of first order differential equations with separating the variables.

Using differential equations to model real-life situations.

Suggested time scale: 20 - 28 lessons

Misconceptions

Mistakes students make when attempting to integrate by substitution include not changing the dx correctly and simply writing it as du, and failing to substitute back to give an expression in x at the end.
Common errors when integrating by parts include: choosing u and dv incorrectly (in particular ln x must always be chosen as u); algebraic errors – especially if they do not remove any common factors to outside the integral sign; incorrect coefficients when integrating dv; and sign errors where sin and cos are involved.
Partial fractions questions are generally done well though some students attempt to integrate the numerator and denominator separately without using partial fractions.
When using the trapezium rule students sometimes mix up the number of strips and the number of x or y values.
The other main place marks are lost is not giving the final answer to three significant figures.
When forming a differential equation some students wrote down the correct differential equation apparently fully understanding all the information given and interpreting it correctly. However, all sorts of errors abounded in other attempts, some not even involving a derivative, and some with derivatives in x and y.
Many had a spurious t and/or h, either as a multiple or power, and the k appeared in a variety of places. Some students did not even form an equation, leaving a proportionality sign in their answer.
When solving a differential equation most students knew they were expected to separate the variables and did it correctly, although there were some notation errors in the positioning of dx, at the front rather than the rear of the integrand. Those who failed to separate the variables, just produced nonsense. Many students struggled with the fact that integration by parts or substitution was needed. All students, no matter what their attempt at the integral, could obtain a method mark if they included a constant and tried to find it using the given initial conditions.

Key Words Tier 2

Replace, identical, adjust, limit.

Key Words Tier 3

Integral, definite integral, indefinite integral, constant of integration, trapezium, substitution, by parts, area, differential equation, first order, separating variables, initial conditions, general solution, parametric.

Homework

Book 2 Unit 12 Mixed exercise
Mathsgenie website: Trigonometric Integration, Exponential Integration, Integration by Substitution, Integration by Parts, Parametric Integration, Differential Equations

Careers links

Financial trader, Purchasing and Quality engineer

Employability skills

Aiming high	Literacy	Creativity	Numeracy
Leadership	Independence	Listening	Communication
Presenting	Teamwork	Problem solving	Staying positive
Critical thinking	Analytical thinking	Time management	

Assessment: **Integration (part 1)** **Integration (part 2)**